Technical Notes

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Integration of Streamlines from Measured Static Pressure Fields on a Surface

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Introduction

FOR three-dimensional boundary-layer calculations, some investigators prefer to know the streamline patterns at the edge of the boundary layer in advance, either to obtain streamline coordinates, to have some guidelines for the choice of a prefixed coordinate system, or to have the components of the freestream velocity for their special computational procedure. Besides these more technical aspects, the streamlines offer a rather good intuitive illustration of the flow and its regions of mutual dependence. For this reason we propose to obtain such a streamline pattern by integrating numerically measured pressure fields using a shooting technique.

Theory

The usual way[1,2] to derive an integration formula is based on two main assumptions:

1) The surface of the body is a stream surface, i.e., the problem is reduced to a two-dimensional one; therefore we have the kinematic condition

$$u^1:u^2 = \mathrm{d}x^1:\mathrm{d}x^2 \tag{1}$$

for a streamline. The angle γ of a streamline relative to a surface coordinate x^σ can be computed from

$$(a_{\alpha\beta}\mathrm{d}x^\alpha \mathrm{d}x^\beta)^{1/2}\cos\gamma = (a_{(\sigma\sigma)})^{1/2}\mathrm{d}x^{(\sigma)} \tag{2}$$

where $a_{\alpha\beta}$ is the metric tensor of the surface and the summation convention is implied.

2) The fluid is inviscid, i.e., the Euler equation for the velocity components u^α, (static) pressure P, and constant density ρ,

$$u^\alpha u^\beta|_\alpha + a^{\alpha\beta}P|_\alpha/\rho = 0 \tag{3}$$

and the Bernoulli equation (for an incompressible fluid),

$$a_{\alpha\beta}u^\alpha u^\beta + P/\rho = \mathrm{const} \tag{4}$$

are valid. As usual, the notation $|_\alpha$ denotes the covariant derivative. The derivation of the Bernoulli equation is based on the assumption that the integration is carried out along a streamline. The flow may be rotational.

Treating the special case of the flow around an inclined ellipsoid (Fig. 1), we introduce the orthogonal coordinate $x = x^1$, that is aligned with the meridians, and $\phi = x^2$, which is measured in the circumferential direction. With r as the local radius of the body perpendicular to the axis and c_p the pressure coefficient, Eqs. (1-4) yield after some manipulations[2]

$$\frac{\mathrm{d}\gamma}{\mathrm{d}x} = \frac{\partial c_p/\partial x\,\tan\gamma - \partial c_p/\partial\phi/r}{2(1-c_p)} - \frac{\mathrm{d}r/\mathrm{d}x}{r}\tan\gamma \tag{5}$$

For a given pressure field $c_p(x,\phi)$, the problem is thus reduced to an initial-value problem for an ordinary differential equation with some starting data for γ at a point on the body.

Recently, Cebeci and Meier[3] applied the Bernoulli equation [Eq. (4)] and the additional assumption that the flowfield is free of rotation to the problem. They obtained streamlines by a two-dimensional numerical scheme taking initial values from the potential solution.

The limits of applicability and feasibility of the integration are due to the incompatibility of both assumptions with the flow of a real fluid. Although assumption 1 postulates connection of the streamlines to the surface, the requirement for an inviscid fluid of assumption 2 can be realized only outside the boundary layer; that is, the boundary-layer thickness is assumed to be zero. Besides the discrepancy caused by the probable variation of static pressure inside the boundary layer, another discrepancy is caused by the different arc lengths of streamlines on the surface and at the edge of the boundary layer when interacting with the pressure field. Notably, both these error sources are neglected in classical boundary-layer theory. Thus the integration should yield fairly correct streamline patterns in the flow regions where the assumption of the classical boundary-layer approximation can be verified.

On the other hand, the boundary-layer computations including Eqs. (3) and (4) for the outer edge also yield streamlines. We recommend this procedure for three-dimensional boundary-layer calculation because the geometric location of these streamlines is more correct.

A great demand on feasibility is the availability of initial values for integration of Eq. (5). Potential solutions of such flows are conventionally used to supply these data near the stagnation point. But the stagnation point of the potential solution differs from those determined from measurements at higher angles of incidence and for more complex flows. As the error of the direction is generally amplified during integration [see Eq. (5)], any deviation of initial values from those corresponding to the real flowfield will deteriorate the results.

To overcome the difficulties of obtaining initial values, we propose another procedure for the determination of streamlines by integration of Eq. (5). General analysis of the topography of a stagnation point[4] for the body under investigation proves that the angle γ between the line of symmetry and a streamline should be either 0 or 180 deg. The other analytically possible values were not taken into account because it is not likely to achieve those streamlines by a numerical procedure.

Only this knowledge about the gross structure was used in this method. From some downstream point on the surface, Eq. (5) was integrated numerically in the upstream direction

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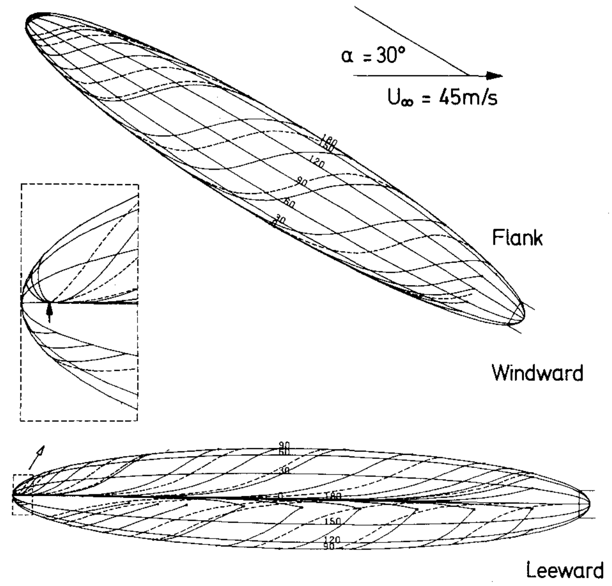

Fig. 1 Geometry of the ellipsoid.

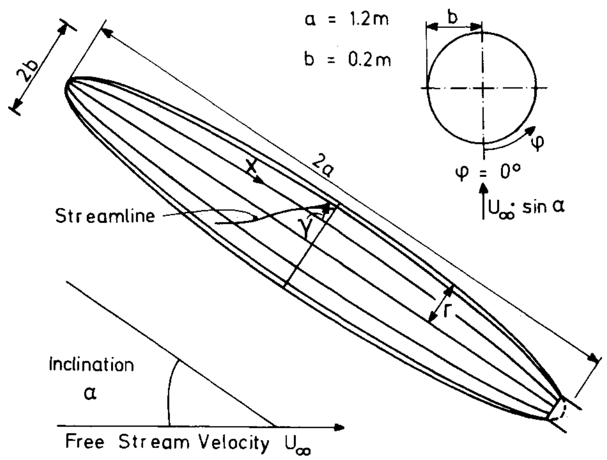

Fig. 2 Streamlines calculated from the static pressure field for $U_\infty = 45$ m/s and $\alpha = 30$ deg (——), and streamlines calculated from the direction field of the velocity components on the surface of the potential solution (--). The arrow (↑) indicates the position of the stagnation point of the potential solution.

toward the expected location of the stagnation point with some estimated initial value of γ at the given point. This integration ended at the line of symmetry with some value of γ that normally deviates from the desired values $\gamma = 0$ or 180 deg. To satisfy this condition for a streamline, the initial value was optimized in such a way that the angle between the integrated trajectory and the line of symmetry became either $\gamma = 0$ or 180 deg. The procedure is equivalent to the shooting technique for solving boundary-value problems of ordinary differential equations. In the treated case, of course, the values could be approached only asymptotically.

Results

The method was applied to the data on static pressure measured by Meier and Kreplin[5] on an axisymmetric ellipsoid. The pressure field of the potential solution was used at the same points for a test. For evaluation of the pressure coefficient and its derivatives, the data were smoothed by means of a spline approximation using a program system supplied by Müller-Wichards.[6]

For the pressure field of the potential solution,[7] the trajectories showed the expected asymptotic behavior. The stagnation point was approached more closely the smaller the values of γ or $180 - \gamma$ deg became, but because of such normal numerical shortcomings as roundoff errors the stagnation point could not be reached. For the measured data asymptotic behavior was observed only down to certain values of x. Then, because of data scattering, a minimum occurred. Therefore trajectories that cut the line of symmetry with values less than 0.4 deg for γ or $180 - \gamma$ deg were accepted as streamlines.

The resulting streamlines for an inclination of $\alpha = 30$ deg and a velocity of $U_\infty = 45$ m/s in Fig. 2 have a rather regular pattern. They are shown with streamlines of the potential solution[7] that have been determined by direct integration of Eq. (1) using a field of line elements given by the components of the velocity on the surface. Near the line of symmetry on the windward side, both streamline patterns have nearly the same direction. For $\phi > 90$ deg the deviation becomes significant, especially on the rear part of the body. Lines are also shown that converge to the line of symmetry on the lee side. The streamlines of both directions could easily be established for a wide range of intersections, demonstrating that the solution of Eq. (5) is not unique to one starting point but depends on boundary conditions. Although these are not the only interesting mathematical features of Eq. (5), due to massive separation of the investigated flowfield the assumptions are not valid in this region and Eq. (5) may not be valid anyway.

For an estimation of the location of the stagnation point, Fig. 2 also shows the enlarged front part of the body. Both groups of lines approaching $\gamma = 0$ or 180 deg, respectively, intersect within less than 1% of the body length. The stagnation point of the potential solution is also found within that interval.

Applying the proposed method to other tests of the axisymmetric ellipsoid of Ref. 4, the method was shown to be feasible if applied with some caution. Within the limitations discussed before, the method may be applied to more complex flowfields if the gross structure is known and an algorithm can be found to describe it.

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